ANGULAR RESOLUTION LIMIT FOR VECTOR-SENSOR ARRAYS: DETECTION AND INFORMATION THEORY APPROACHES

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ABSTRACT

The Angular Resolution Limit (ARL) on resolving two closely spaced polarized sources using vector-sensor arrays is considered in this paper. The proposed method is based on the information theory. In particular, the Stein's lemma provides, asymptotically, a link between the probability of false alarm and the relative entropy between two hypothesis of a given statistical binary test. We show that the relative entropy can be approximated by a quadratic function in the ARL. This property allows us to derive and analyze a closed-form expression of the ARL. To illustrate the interest of our approach, the ARL, in the sense of the detection theory, is also derived. Finally, we show that the ARL is only sensitive to the norm of the polarization state vector and not to the particular values of the polarization parameters.

Index Terms— Angular resolution limit, polarized source localization, distance measure, information theory, detection theory.

1. INTRODUCTION

The Direction-Of-Arrivals (DOA) estimation for polarized sources based on the vector-sensor arrays have been largely investigated in the last decade. In [1], the authors considered the problem of source localization in a multipath environment by using a vector-sensor array consisting of three electric and three magnetic dipoles. In [2], a tensorial version of the MUSIC algorithm for vector-sensor arrays was presented. On the other hand, the polarized seismic wave estimation was considered in [3]. Whereas, in [4], closed form expressions of Cramér-Rao Bound (CRB) have been derived.

In array processing, the Angular Resolution Limit (ARL) characterizes the minimum angular separation to resolve two closely spaced sources. In the literature, there are three approaches to obtain the ARL. (1) The first approach based on the estimation accuracy. In this way, Smith [5] proposed the following criterion based on the CRB: two signals are resolvable if the separation between the two DOAs θ_1 and θ_2 is less than the standard deviation of the separation estimation. Consequently, the ARL in the Smith sense is defined as the angular separation between the parameters of interest that is equal to the standard deviation of the angular separation. Furthermore, in [6], the extension of the ARL in the case of multiple parameters per signal based on Smith criteria was presented. (2) The second approach based on the concept of the mean null spectrum [7]. This approach is quite intuitive but is only relevant to a specific high-resolution algorithm. (3) The third approach based on detection theory. In [8], the ARL based on the Minimum Probability of Error (MPE) for the deterministic signals is investigated. The authors used the first order Taylor expansion of the MPE to derive the ARL. Whereas, in [9] Liu and Nehorai have defined the statistical angular resolution limit using the asymptotic equivalence of the Generalized Likelihood Ratio Test (GLRT). On the other hand, Sharman and Milanfard [10] derived the frequency resolution limit in the spectral analysis using the GLRT.

In this paper, we consider the approach of the ARL based on the information theory, and more specifically on the Stein's lemma [11]. The Stein's lemma links the false alarm probability (P_{fa}) resulted from the Neyman-Pearson decision criterion to the relative entropy (also called Kullback-Leiber pseudo-distance). As the relative entropy can be approximated by a quadratic function in the ARL, it is possible to determine the ARL by this way. To illustrate our approach, the ARL is derived in the Neyman-Pearson sense in the context of the detection theory. Finally, we also compare our approach to the one presented by Amar and Weiss in reference [8].

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2. MODEL SETUP

2.1. Polarized signal model

We consider the context of DOA estimation of two narrow-band polarized source signals using a linear two elements vector-sensor array. Without loss of generality, we assume that the array lays on the Ox axis of the Cartesian coordinate. The array consists of N vector-sensors and the known positions of these vector-sensors in the array are given by the vector $\mathbf{d} = [d_1 \ldots d_N]^T$. The two sources are assumed to be located in the far-field, deterministic, and coplanar with the array, *i.e.*, the elevation angles of the sources equal $\theta_k = \pi/2$, $k = \{1, 2\}^2$. For mathematical convenience, we consider the estimation of $u_k = \frac{2\pi}{\lambda} \sin \phi_k$, where ϕ_k denotes the azimuth angles of the k^{th} source, and where λ denotes the wavelength.

We assume that the source polarization is constant in time and along the array. The polarization of the sources is characterized by the vector [2]

$$\mathbf{p}(\rho, \varphi) = \begin{bmatrix} [\mathbf{p}(\rho, \varphi)]_1 \\ [\mathbf{p}(\rho, \varphi)]_2 \end{bmatrix} = \frac{1}{\sqrt{\rho^2 + 1}} \begin{bmatrix} 1 \\ \rho e^{j\varphi} \end{bmatrix}, \qquad (1)$$

where ρ and φ denote the amplitude ratio and the phase shift between the second component of the sensor and the first [2]. At the t^{th} snapshot, the output signal in the time domain at the i^{th} vector-sensor consists of two components (the model in frequency domain is available in [2])

$$\hat{z}_i(t) = \sum_{k=1}^{2} [\mathbf{p}(\rho,\varphi)]_1 \, [\mathbf{a}(u_k)]_i s_k(t) + \hat{n}_i(t) \tag{2}$$

$$\breve{z}_i(t) = \sum_{k=1}^{2} [\mathbf{p}(\rho,\varphi)]_2 \, [\mathbf{a}(u_k)]_i s_k(t) + \breve{n}_i(t) \tag{3}$$

where $\mathbf{a}(u_k) = \begin{bmatrix} e^{-jd_1 u_k} & \ldots & e^{-jd_N u_k} \end{bmatrix}^T$, and where $\breve{n}(t)$, $\hat{n}(t)$ denote the additive noises at the t^{th} snapshot, and $s_k(t)$ denotes the source signal of the k^{th} source at the t^{th} observation. We assume that $s_1(t) \neq s_2(t), \forall t$. The noises are assumed to be complex, circular, white Gaussian with zero mean and covariance matrix $\sigma^2\mathbf{I}$, *i.e.*, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$. Furthermore, we assumed the signal sequences of the two sources are deterministic with known sequences at the observer. Consequently, at the t^{th} snapshot, the response vector $\mathbf{z}(t) = [\hat{z}_1(t) \, \ldots \, \hat{z}_N(t) \; \breve{z}_1(t) \, \ldots \, \breve{z}_N(t)]^T$ of the array is given by

$$\mathbf{z}(t) = \sum_{k=1}^{2} \mathbf{b}(u_k) s_k(t) + \mathbf{n}(t) \tag{4}$$

where $\mathbf{n}(t)$ denotes the noise vector and $\mathbf{b}(u_k) = \mathbf{p}(\rho,\varphi) \otimes \mathbf{a}(u_k)$ in which $\otimes$ stands for the Kronecker product.

2.2. MPE based binary hypothesis test

We can now adopt the two detection hypotheses (see [8]): under hypothesis $\mathcal{H}_0$, the observer detects only a single signal, which is a combination of the two sources, and under $\mathcal{H}_1$, the observer detects two signals:

$$\begin{cases} \mathcal{H}_0: & \mathbf{z}(t) = \mathbf{b}(\hat{u}(t))\hat{s}(t) + \mathbf{n}(t), \\ \mathcal{H}_1: & \mathbf{z}(t) = \sum\limits_{k=1}^{2} \mathbf{b}(u_k) s_k(t) + \mathbf{n}(t) \end{cases} \tag{5}$$

where $\hat{u}(t)$ and $\hat{s}(t)$ denote the parameter and signal amplitude under $\mathcal{H}_0$, whereas the probability of error P_e given by $P_e = p(\mathcal{H}_0)P_{fa} + p(\mathcal{H}_1)P_m$, in which P_{fa} and P_m denote the probability of false alarm and the probability of miss, respectively, and where $p(\mathcal{H}_0)$, $p(\mathcal{H}_1)$ denote the prior probability of the two hypotheses. Without loss of generality, we assume that $p(\mathcal{H}_0) = p(\mathcal{H}_1) = 1/2$. Thus, setting $u_c = \frac{u_1+u_2}{2}$ and the ARL given by $\delta = u_2 - u_1$, then, the values of $\hat{u}(t)$, and $\hat{s}(t)$ chosen according to the minimal probability of error (MPE) criteria are given by [8]:

$$\hat{u}(t) = u_c + \gamma(t)\delta, \tag{6}$$

$$\hat{s}(t) = \frac{1}{2N} \mathbf{b}^H(\hat{u}(t)) \left(\mathbf{b}(u_c - \frac{\delta}{2}) s_1(t) + \mathbf{b}(u_c + \frac{\delta}{2}) s_2(t) \right) \tag{7}$$

where $\gamma(t) = \frac{|s_2(t)|^2 - |s_1(t)|^2}{2\left(|s_2(t)|^2 + |s_1(t)|^2 + 2\Re\left\{s_1^*(t)s_2(t)\right\}\right)}$. From the aforementioned assumption, it is clear that

$$\begin{cases} \mathcal{H}_0: & \mathbf{z}(t) \sim \mathcal{CN}(\boldsymbol{\mu}_0(t), \sigma^2\mathbf{I}), \text{ where } \boldsymbol{\mu}_0(t) = \mathbf{b}(\hat{u}(t))\hat{s}(t), \\ \mathcal{H}_1: & \mathbf{z}(t) \sim \mathcal{CN}(\boldsymbol{\mu}_1(t), \sigma^2\mathbf{I}) \text{ where } \boldsymbol{\mu}_1(t) = \sum\limits_{k=1}^{2} \mathbf{b}(u_k) s_k(t). \end{cases} \tag{8}$$

2.3. First-order Taylor expansions

Assume that the polarization parameters are all equal[1], *i.e.* $\rho_1 = \rho_2 = \rho_0$ and $\varphi_1 = \varphi_2 = \varphi_0$. The first-order Taylor expansions of the vectors of interests are considered as follows:

$$\mathbf{b}(u_1) = \mathbf{b}(u_c) - \frac{\delta}{2}\dot{\mathbf{b}}(u_c) \;\text{ at } (u_1 = u_c - \tfrac{\delta}{2}, \rho_0, \varphi_0),$$

$$\mathbf{b}(u_2) = \mathbf{b}(u_c) + \frac{\delta}{2}\dot{\mathbf{b}}(u_c) \;\text{ at } (u_2 = u_c + \tfrac{\delta}{2}, \rho_0, \varphi_0),$$

$$\mathbf{b}(\hat{u}(t)) = \mathbf{b}(u_c) + \gamma(t)\delta\dot{\mathbf{b}}(u_c) \;\text{ at } (u'(t) = u_c + \gamma(t)\delta, \rho_0, \varphi_0)$$

in which the first-order derivative *w.r.t.* u_c of vector $\mathbf{b}(u_c) = \mathbf{p}(\rho_0,\varphi_0) \otimes \mathbf{a}(u_c)$ is defined as $\dot{\mathbf{b}}(u_c) = \mathbf{p}(\rho_0,\varphi_0) \otimes \dot{\mathbf{a}}(u_c)$ where $\dot{\mathbf{a}}(u_c) = j\text{diag}\{\mathbf{d}\}\mathbf{a}(u_c)$. So, the optimal value (7) becomes

$$\hat{s}(t) \cong p(t) + \frac{\delta}{2N}\kappa_c m(t) \tag{9}$$

in which $p(t) = s_1(t) + s_2(t)$, $\kappa_c = \mathbf{b}^H(u_c)\dot{\mathbf{b}}(u_c)$, $\mathbf{m} = [m(1) \ldots m(T)]^T = \mathbf{V}^T\mathbf{s}$ where $\mathbf{s} = [s_1(1) \; s_2(1) \ldots s_1(T) \; s_2(T)]^T$ and $\mathbf{V} = \text{Bdiag}\{\mathbf{v}(1), \ldots, \mathbf{v}(T)\}$ with $\mathbf{v}(t) = [\gamma(t) + \frac{1}{2} \;\; \gamma(t) - \frac{1}{2}]^T$. According to the previous expression, we can see that the optimal source $\hat{s}(t)$ is approximated by a linear combination of the sources $s_1(t)$ and $s_2(t)$. Consequently, using the above expression the first-order Taylor expansion of the mean under $\mathcal{H}_0$ and $\mathcal{H}_1$ can be rewritten as

$$\boldsymbol{\mu}_0(t) = \mathbf{b}(\hat{u}(t))\hat{s}(t) \cong \delta\boldsymbol{\nu}_0(t), \tag{10}$$

$$\boldsymbol{\mu}_1(t) = \sum_{k=1}^{2} \mathbf{b}(u_k) s_k(t) \cong \delta\boldsymbol{\nu}_1(t). \tag{11}$$

where

$$\boldsymbol{\nu}_0(t) = p(t)\gamma(t)\dot{\mathbf{b}}(u_c) + \left(\frac{\kappa_c m(t)}{2N}\right)\mathbf{b}(u_c) \tag{12}$$

$$\boldsymbol{\nu}_1(t) = \frac{q(t)}{2}\dot{\mathbf{b}}(u_c). \tag{13}$$

in which $q(t) = s_2(t) - s_1(t)$. So, the linearized hypothesis test is given by

$$\begin{cases} \mathcal{H}_0: & \mathbf{z} \cong \delta\boldsymbol{\nu}_0 + \mathbf{n}, \\ \mathcal{H}_1: & \mathbf{z} \cong \delta\boldsymbol{\nu}_1 + \mathbf{n} \end{cases} \tag{14}$$

where $\mathbf{z} = [\mathbf{z}(1)^T \ldots \mathbf{z}(T)^T]^T$, $\mathbf{n} = [\mathbf{n}(1)^T \ldots \mathbf{n}(T)^T]^T$, $\boldsymbol{\nu}_0 = \left[\boldsymbol{\nu}_0^T(1) \ldots \boldsymbol{\nu}_0^T(T)\right]^T$ and $\boldsymbol{\nu}_1 = \left[\boldsymbol{\nu}_1^T(1) \ldots \boldsymbol{\nu}_1^T(T)\right]^T$.

3. ARL BASED ON INFORMATION THEORY

3.1. Stein's lemma

By maximizing the probability of detection (*i.e.* $P_d \approx 1$) for $P_{fa} \leq \epsilon$ with ϵ goes to zero slowly, the best error exponent resulting from using Neyman-Pearson test is given by the Stein's lemma [11,12] as follows:

$$\lim_{T\to\infty} \ln P_{fa} = -\mathcal{D}(p(\mathbf{z}|\mathcal{H}_1) \| p(\mathbf{z}|\mathcal{H}_0)), \tag{15}$$

[1]Note that this situation is the worst case in the resolution point of view.

where $\mathcal{D}(p(\mathbf{z}|\mathcal{H}_1)\|p(\mathbf{y}|\mathcal{H}_0))$ denotes the relative entropy. Let Ω be the observation space. After some trivial calculations, the relative entropy between two Gaussian distributions with parameterized means is straightforwardly given by:

$$\mathcal{D}(p(\mathbf{z}|\mathcal{H}_1)\|p(\mathbf{z}|\mathcal{H}_0)) = \int_{\Omega} p(\mathbf{z}|\mathcal{H}_1) \ln\left(\frac{p(\mathbf{z}|\mathcal{H}_1)}{p(\mathbf{z}|\mathcal{H}_0)}\right) d\mathbf{z} = \frac{1}{\sigma^2}\sum_{t=1}^{T} \|\boldsymbol{\mu}_0(t) - \boldsymbol{\mu}_1(t)\|^2 .$$

3.2. Geometrical expression of the relative entropy

Using relations (10) and (11), we can link the relative entropy and the ARL according to

$$\begin{aligned}\mathcal{D}(p(\mathbf{y}|\mathcal{H}_1)\|p(\mathbf{y}|\mathcal{H}_0)) &\cong \frac{\delta^2}{\sigma^2}\sum_{t=1}^{T} \|\boldsymbol{\nu}_0(t) - \boldsymbol{\nu}_1(t)\|^2 \\ &= \frac{\delta^2||\mathbf{m}||^2}{\sigma^2}\left\|\frac{\kappa_c}{N}\mathbf{b}(u_c) - \dot{\mathbf{b}}(u_c)\right\|^2 \\ &= \frac{\delta^2||\mathbf{m}||^2||\mathbf{d}||^2}{\sigma^2}\cos^2(\Theta)\end{aligned}$$

using $||\mathbf{p}(\rho,\varphi)||^2 = 1$, $||\mathbf{a}(u_c)||^2 = N$, $||\dot{\mathbf{a}}(u_c)||^2 = ||\mathbf{d}||^2$ and Θ is the largest canonical angle between vectors $\mathbf{a}(u_c)$ and $\dot{\mathbf{a}}(u_c)$. The important point is that the relative entropy can be approximated by a quadratic (in δ) expression. In addition, the relative entropy is a function of the source waveforms, of the array distribution, of the noise variance and of the a useful geometrical quantity which is the "angle" between the steering vector and its first-order derivative. It is interesting to note that the minimal value of $1/\cos^2(\Theta)$ which is reached for collinear vectors (*i.e.* $\Theta = 0$) is not relevant since by construction the steering vector and its first-order derivative cannot be collinear. Another geometrical interpretation is: the more orthogonal the two vectors, the smaller the relative entropy. This means that it could be more and more difficult to discriminate the two hypothesis. According to the expression of the relative entropy, we can see that to ensure a "good" discrimination of the two hypothesis, we must have a large ARL or/and a large array distribution and/or a small noise variance.

3.3. ARL based on the Stein's lemma

Thus, from the above expressions and using the fact that for optimal P_d (close to one), $P_{fa} \approx 2P_e = 2(1-\eta)$, the ARL takes a closed-form expression according to

$$\delta \cong \frac{-\sigma\sqrt{\log(2) + \log(1-\eta)}}{\mu\,||\mathbf{d}||\cos(\Theta)} \tag{16}$$

where $\eta > 1/2$, $\mu = \sqrt{\mathbf{s}^H\mathbf{V}\mathbf{V}^T\mathbf{s}}$ in which

$$\mathbf{V}\mathbf{V}^T = \text{Bdiag}\{\mathbf{G}(1)\ldots\mathbf{G}(T)\}$$

where

$$\mathbf{G}(t) = \begin{bmatrix} \left(\gamma(t)+\frac{1}{2}\right)^2 & \gamma^2(t)+\frac{1}{4} \\ \gamma^2(t)+\frac{1}{4} & \left(\gamma(t)-\frac{1}{2}\right)^2\end{bmatrix}. \tag{17}$$

It is interesting to note that the ARL is affected by the waveform design, (*cf.* quantity μ) but not from the polarization state in case of unit norm of the polarization vector.

4. DETECTION THEORY APPROACH

In this Section, we derive the ARL using the detection theory approach, particularly, using the well-known Neyman-Pearson (NP) criterion. The NP will minimize the probability of error P_e. Even if the proposed approach concerns the minimization of P_e as the one presented by Amar and Weiss [8], our approach is different. Indeed, Amar and Weiss derive the ARL, denoted Theoretical Resolution Limit (TRL), based on the linearization of the error probability. In our method, we choose to linearize directly the observation signal as done by Sharman and Milanfard [10].

In order to simplify the calculation, we perform the following change of variable formula:

$$\mathbf{z}' = \frac{\mathbf{z}}{\delta} - \boldsymbol{\nu}_0. \tag{18}$$

Consequently, plugging (18) into (14), one obtains

$$\begin{cases}\mathcal{H}_0: & \mathbf{z}' \cong \mathbf{n}' \\ \mathcal{H}_1: & \mathbf{z}' \cong \boldsymbol{\zeta} + \mathbf{n}'\end{cases} \tag{19}$$

where $\boldsymbol{\zeta} = \boldsymbol{\nu}_1 - \boldsymbol{\nu}_0$ and $\mathbf{n}' \sim \mathcal{CN}(\mathbf{0}, \frac{\sigma^2}{\delta^2}\mathbf{I})$. Consequently, one has

$$G_{NP}(\mathbf{z}') = \frac{p(\mathbf{z}'|\mathcal{H}_1)}{p(\mathbf{z}'|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau' = \frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}, \tag{20}$$

denoting $T_{NP}(\mathbf{z}') = \ln(G_{NP}(\mathbf{z}'))$ and $\tau = \ln(\tau')$, the statistics test can be given by

$$\begin{aligned}T_{NP}(\mathbf{z}') = \ln\left(\frac{p(\mathbf{z}'|\mathcal{H}_1)}{p(\mathbf{z}'|\mathcal{H}_0)}\right) &= \frac{\delta^2}{\sigma^2}\left(\|\mathbf{z}'-\boldsymbol{\zeta}\|^2 - \|\mathbf{z}'\|^2\right) \\ &= \frac{\delta^2}{\sigma^2}\left(\|\boldsymbol{\zeta}\|^2 - 2\Re\left\{\mathbf{a}^H\mathbf{z}'\right\}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau.\end{aligned} \tag{21}$$

Since we have assumed that $p(\mathcal{H}_0) = p(\mathcal{H}_1) = 1/2$, one obtains

$$\begin{cases}\mathcal{H}_0: & T_{NP}(\mathbf{z}') > 0 \\ \mathcal{H}_1: & T_{NP}(\mathbf{z}') < 0.\end{cases} \tag{22}$$

Let $L(\mathbf{z}') = \Re\left\{\boldsymbol{\zeta}^H\mathbf{z}'\right\}$, one can easily obtain

$$\begin{cases}\mathcal{H}_0: & L(\mathbf{z}') \sim \mathcal{N}(0, \varrho^2) \\ \mathcal{H}_1: & L(\mathbf{z}') \sim \mathcal{N}(\|\boldsymbol{\zeta}\|^2, \varrho^2)\end{cases} \tag{23}$$

where

$$\varrho^2 = \frac{\sigma^2\|\boldsymbol{\zeta}\|^2}{2\delta^2}.$$

Thus [13],

$$P_e = \frac{1}{2}\left(\left(1 - Q\left(\frac{-\|\boldsymbol{\zeta}\|^2}{2\sqrt{\varrho^2}}\right)\right) + Q\left(\frac{\|\boldsymbol{\zeta}\|^2}{2\sqrt{\varrho^2}}\right)\right), \tag{24}$$

in which $Q(.)$ denotes the right-tail function of the probability function for a Gaussian random variable with zero mean and unit variance. Since $Q\left(\frac{-\|\boldsymbol{\zeta}\|^2}{2\sqrt{\varrho^2}}\right) = 1 - Q\left(\frac{\|\boldsymbol{\zeta}\|^2}{2\sqrt{\varrho^2}}\right)$, thus, one obtains

$$P_e = Q\left(\frac{\|\boldsymbol{\zeta}\|^2}{\sqrt{4\varrho^2}}\right) \tag{25}$$

Consequently, the ARL based on the NP criteria is given by

$$\delta \cong \frac{\sigma\sqrt{2}Q^{-1}(1-\eta)}{\|\mathbf{m}\|\,\|\mathbf{d}\|\cos(\Theta)} \tag{26}$$

where $Q^{-1}(.)$ is the inverse of the right-tail function of the probability function for a Gaussian random variable with zero mean and unit variance.

5. SIMULATIONS RESULTS

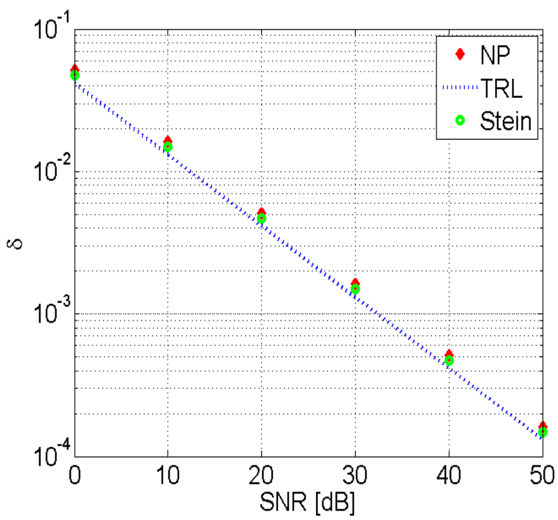

Fig. 1. ARL vs. SNR. The considered ARL are based on the Information Theory, on the Neyman-Pearson (NP), and on reference [8] denoted by TRL for Theoretical Resolution Limit.

In this section, we will compare the ARL based on the Information Theory, on the Neyman-Pearson (NP), and on reference [8] denoted by TRL for Theoretical Resolution Limit. We consider an uniform linear array consisting of $N = 10$ vector-sensor components with a half-wavelength equidistantly space. The polarization parameters were chosen as $\rho_0 = 1$ and $\varphi_0 = \pi/3$. The success rate is $\eta = 0.99$ thus P_e is set to be 0.01, and the probability of false alarm equals $P_{fa} = 0.02$. We consider a large number of observations $T = 100$. Since all parameters are herein assumed to be deterministic, the expectation concerning the TRL in [8], will not be performed. The signal to noise ratio SNR is given by $SNR = \left(\sum_{k=1}^{2}\|\mathbf{s}_k\|^2\right) / \left(2T\sigma^2\right)$. Fig. 1 shows the behavior of the ARL versus the SNR. One observes that the ARL based on the three methods are very close. It is interesting to note that the ARL is affected by the waveform design but not from the polarization state in case of unit norm of the polarization vector.

6. CONCLUSION

We have introduced, in this paper, a new approach based on the information theory to obtain the ARL for two closely-spaced polarized sources using a vector-sensor array. The key point is the fact that the relative entropy, in the Stein's lemma, can be approximated by a quadratic function in the ARL. So, it is possible to derive the ARL following this methodology. In addition, a geometrical expression of the ARL is provided. Finally, we compare our approach to the Neyman-Pearson test (also derived in this paper) and to the Theoretical Resolution limit (TRL) proposed by Amar and Weiss. In particular, we show that for all these methods, the ARL is a function of the (unit) norm of the polarization vector but not of the specific values of the polarization parameters (under the assumption that the polarizations parameters are all equal).

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