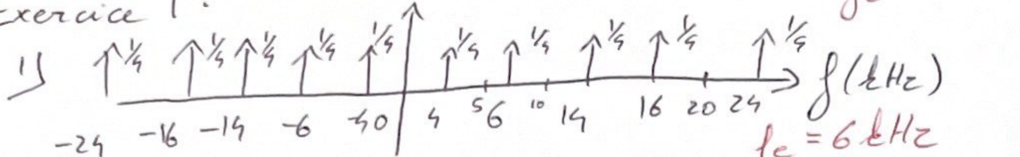
Partiel Signal UE305 L3E3A 2024 ①

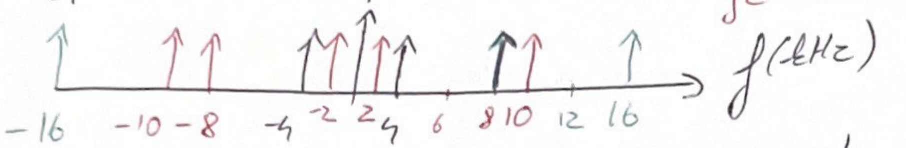
Exercice 1 :

$f_e = 10\,kHz$

1)



$f_e = 6\,kHz$



2) $a_1 = 1$; $b_1 = 0$; $a_3 = -1$; $b_3 = 1$ $\Rightarrow$ $c_1 = \frac{a_1 - i b_1}{2} = \boxed{\frac{1}{2}}$; $c_{-1} = \frac{a_1 + i b_1}{2} = \boxed{\frac{1}{2}}$

$F_{MAX} = \frac{3 \times 100 \times \pi}{2\pi} = 150\,Hz$ $\Rightarrow c_3 = \boxed{-\frac{(1+i)}{2}}$ et $c_{-3} = \boxed{\frac{1-i}{2}}$

$\Rightarrow F_e \geqslant 2 F_{MAY} = \boxed{300\,Hz}$ (+ fréquence de $s(t)$: $\omega = 100\pi \Rightarrow f = \frac{100\pi}{2\pi} = \boxed{50\,Hz}$)

3) • 1 période dure $\frac{1}{f}$ secondes $\Rightarrow$ en 1 sec on a $\boxed{f \text{ périodes}}$

• pour 1 période, on a $\frac{1/f}{1/F_e}$ points $= \frac{F_e}{f}$ $\Rightarrow$ pour $\Big|$ 1 sec / f période on a $\boxed{F_e \text{ points}}$

$= \boxed{1/d}$

• $F_e \geqslant 2 f_{MAX}$ $\Rightarrow$ ici $F_e \geqslant 2f \Rightarrow \frac{f}{F_e} \leqslant \frac{1}{2}$ $\Leftrightarrow$ $\boxed{d \leqslant \frac{1}{2}}$

4) Il faut $F_e \geqslant 2F_{MAX}$. Ici $F_{MAX} = \frac{320\pi}{2\pi} = 160\,Hz$ $\Rightarrow$ Shannon n'est pas respecté

$F_e = 140\,Hz$

Exercice 2 :

1) $\hat{x}(f) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-i2\pi f n} = e^{-i2\pi f 0} + e^{-i2\pi f \times 2} = 1 + e^{-i4\pi f}$

$= 2\cos(2\pi f)\, e^{-2i\pi f}$

2) $x[n] = \int_{-1/2}^{1/2} \hat{x}(f)\, e^{i2\pi f n}\, df = \int_{-1/2}^{1/2} (1 + e^{-i4\pi f})\, e^{i2\pi f n}\, df$

Si $n = 0$: $\int_{-1/2}^{1/2} e^{i2\pi f n}\, df = 1$; Si $n \neq 0$ $\int_{-1/2}^{1/2} e^{i2\pi f n}\, df = \frac{e^{i\pi n} - e^{-i\pi n}}{i2\pi n} = 0$

Si $n \neq 2$: $\int_{-1/2}^{1/2} e^{i2\pi(n-2)f}\, df = \frac{e^{i\pi(n-2)} - e^{-i\pi(n-2)}}{i2\pi(n-2)} = 0$

Si $n = 2$: $\int_{-1/2}^{1/2} e^{i2\pi(n-2)f}\, df = 1$

3) $1 + y + y^2 = \frac{1 - y^3}{1 - y}$ car $(1-y)(1+y+y^3) = 1 - y^3$

si on pose $y = e^{i\theta}$ $\Rightarrow 1 + y + y^2 = y\, \frac{y^{-3/2} - y^{3/2}}{y^{-1/2} - y^{1/2}} = e^{i\theta}\, \frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$

$\hat{y}(f) = \sum_{n=-\infty}^{+\infty} y[n]\, e^{-i2\pi f n} = 1 + e^{-i2\pi f} + e^{-i4\pi f} = e^{i2\pi f}\, \frac{\sin\left(-\frac{3}{2} 2\pi f\right)}{\sin\left(-\frac{1}{2} 2\pi f\right)}$

$= e^{i2\pi f}\, \frac{\sin 3\pi f}{\sin(\pi f)}$

Exercice 3

1) l'équation E/S s'écrit $i_E(t) = i_R(t) + i_C(t) + i_L(t)$

=> il s'agit d'opérateurs linéaires $= R\,u(t) + C\frac{du(t)}{dt} + \frac{1}{L}\int u\,dt$

2) $CI = 0 \Rightarrow I_E(p) = \frac{U_E(p)}{R} + Cp\,U_E(p) + \frac{1}{Lp}U_E(p)$

$\Rightarrow I_E(p) = \boxed{\frac{1+\frac{L}{R}p + CLp^2}{Lp}}\,U_E(p)$

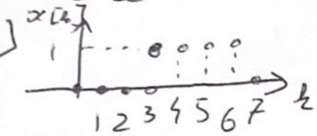
3) Si $R \to \infty$ et $I_E(p) = 1$ alors $U_E(p) = \frac{Lp}{1+CLp^2}$

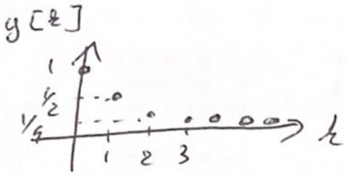
D'après la table des TL : $TL\left\{\frac{p}{p^2+\omega_0^2}\right\} = \sin(\omega_0 t)\,H(t)$

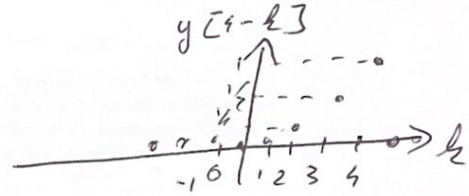
Ici $\frac{Lp}{1+CLp^2} = \frac{L}{CL}\,\frac{p}{p^2+\frac{1}{CL}} = \frac{1}{C}\,\frac{p}{p^2+\omega_0^2} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

Exercice 4 :

1)







2) $\sum_{k=m}^{n} q^k = q^m + q^{m+1} + \dots + q^n \Rightarrow \frac{1}{q^m}\sum_{k=m}^{n} q^k = 1 + q + q^2 + \dots + q^{n-m}$

$\Rightarrow \frac{1-q}{q^m}\sum_{k=m}^{n} q^k = 1 + \cancel{q} + \cancel{q^2} + \dots + \cancel{q^{n-m}} - \cancel{q} - \cancel{q^2} - \dots - q^{n-m+1}$

3) Si $k < 3$ $x[l]$ et $y[k-l]$ n'ont pas de nb en commun $\Rightarrow y[k] = 0$

Si $k \geq 6$ alors $y[k] = \sum_{l=3}^{6} (1/2)^{k-l} = \left(\frac{1}{2}\right)^k \cdot \underbrace{2^3}_{8} \cdot \frac{1-2^4}{1-2} = \boxed{120 \cdot 2^{-k}}$

Si $3 \leq k \leq 6$ alors $y[k] = \sum_{l=3}^{k} (1/2)^{k-l} = \left(\frac{1}{2}\right)^k \cdot 2^3 \cdot \frac{1-2^{k-3+1}}{1-2}$

$= \boxed{2(1 - 4 \cdot 2^{-k})}$

Exercice 5 :

On sait que $TF\{\Pi_T(t)\} = \frac{\sin(\pi f T)}{\pi f}$. Ici $x(t) = 2\,\Pi_2(t) + \Pi_2(t-4) + \Pi_2(t+4)$

$\Rightarrow \hat{X}(f) = 2\,\frac{\sin(2\pi f)}{\pi f} + \frac{\sin(2\pi f)}{\pi f}\underbrace{\left(e^{-2i\pi f \times 4} + e^{2i\pi f \times 4}\right)}_{= 2\cos(8\pi f)}$

$\hat{X}(f) = \boxed{\frac{2\sin(2\pi f)}{\pi f}\left(\cos(8\pi f) + 1\right)}$ et $\boxed{\hat{X}(0) = 8}$

Exercice 6 :

1) $s[0] = 2\underbrace{s[-1]}_{\substack{=0 \\ \text{(sortie causale)}}} + \underbrace{e[0]}_{=1} = 1$; $s[1] = e[1] + 2s[0] = 3$; $s[2] = e[2] + 2s[1] = 7$

2) $y[n] = s[n] + 1 \Rightarrow y[n+1] = s[n+1] + 1 = 2s[n] + 1 + 1 = \overset{\text{raison}}{2}\,y[n]$

$$\Rightarrow y[n] = \underbrace{2\times 2\times 2\times\cdots\times 2}_{n\ \text{fois}}\ \underbrace{y[0]}_{=2} \Rightarrow \boxed{y[n] = 2^{n+1}}$$

3) $s[n] = y[n] - 1 = \boxed{2^{n+1} - 1} \Rightarrow s[0] = 1 ; s[1] = 3$ et $s[2] = 7$

4) $s[n] = 2s[n-1] + e[n] \Rightarrow S(z) - 2z^{-1}S(z) = \underbrace{E(z)}_{= z/(z-1)}$

$$\Rightarrow S(z) = \frac{z/(z-1)}{1 - 2/z} = \frac{z}{z-1} \times \frac{z}{z-2} = \frac{z^2}{(z-1)(z-2)}$$

5) $$\frac{S(z)}{z} = \frac{z}{(z-1)(z-2)} = -\frac{1}{z-1} + \frac{2}{z-2}$$

6) $$S(z) = -\frac{z}{z-1} + \frac{2z}{z-2} \Rightarrow s[n] = (2 \times 2^n - 1)H(n) = (2^{n+1} - 1)H(n)$$

Exercice 7 :

1) $\boxed{a_0 = \frac{\pi^2}{3}}$ $\boxed{a_n = \frac{4(-1)^n}{n^2}}$ et $\boxed{b_n = 0}$

fct paire

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2\,dt = \frac{1}{2\pi}\left[\frac{t^3}{3}\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 \cos nt\,dt$$

$\hookrightarrow$ double IPP sur t^2

2) $$\sum_{n=1}^{+\infty} \frac{1}{n^4} = \boxed{\frac{\pi^4}{90}}$$